Math 3010 - Calculus II: Further Study of Motion and Change

Instructor: Ross Parker (rparker@sarahlawrence.edu)
Seminar hours: Tu/Th 3:35 - 5:00 pm
Conference hours: 30 minutes biweekly, times to be arranged
Course location: Library E2
Conference (office) location: Dudley Lawrence 2B
(2nd floor, up staircase on side of building closest to Westlands)

Course Description

This course continues the thread of mathematical inquiry, following an initial study of the dual topics of differentiation and integration (covered in a Calculus I course). Topics to be explored in this course include the calculus of exponential and logarithmic functions, applications of integration theory to geometry, alternative coordinate systems, infinite series, and power series representations of functions. For conference work, students may choose to undertake a deeper investigation of a single topic or application of the calculus or conduct a study of some other mathematically-related topic, including artistic projects. This seminar is intended for students interested in advanced study in mathematics or science, preparing for careers in the health sciences or engineering, or simply wishing to broaden and enrich the life of the mind.

Prerequisites

Successful completion of Calculus I, or equivalent.

Learning goals

- Determine convergence of sequences and infinite series.
- Use Taylor series to approximate functions with polynomials.
- Learn how to compute the integral of a function using techniques such as substitution and integration by parts.
- Apply integration theory to geometric problems in rectangular, parametric, or polar form.
- Set up and solve first order differential equations which model problems in the natural and social sciences.
- Explore in depth a topic of your own choosing in conference. Communicate your work in both a written report and a presentation to the entire class.

Seminar

This course is a **small seminar**. We will meet twice per week, with 85 minutes per class. Although I like to think that I'm entertaining, I will not be lecturing. (Turns out that's not a great way to learn math!) Instead we will be exploring the course material collaboratively. The class time will include (but not be limited to) the following:

- I present new material on the board. I will limit this as much as possible. Please feel free to interrupt me at any time to ask me to repeat something, clarify a point, request I slow down, fix my handwriting, point out a mistake I have made, etc.
- You will do problems in class, either individually or in small groups.
- You will work through problems on the board. Students will be chosen uniformly at random to do this, but you will be assisted by the rest of the class (and me). Some of the problems may be ones you have prepared as part of your assignment due that day. Some might not.
- You will visualize mathematical concepts using online resources such as Desmos and Wolfram Alpha.

Within the confines of the material I am required to cover, I will encourage the class to explore topics of mutual interest... as long as it involves math.

Conference

You will meet one-on-one with me for 30 minutes biweekly in conference. This is an integral (pun intended!) part of the course, and attendance at conference is **required**. Conference will consist of the following:

- We go over your written assignments that I have collected since the previous conference.
- We discuss your progress in the course: what topics you understand, what topics could use clarification, etc. We make a plan to address areas that need improvement.
- We discuss your progress on the course project. We review the goals you set the previous conference, you present your progress on those goals, and we set goals for the next conference.

Attendance

Attendance in seminar and conference, as well as the timely submission of assignments, is **required** in this course. Your success in this class depends entirely on regular attendance in class and conference, the regular and timely completion of your homework, and your final project. Students who miss more than two classes or conferences run the risk of receiving reduced course credit and/or a lower grade in the course, and this will also be noted on your narrative evaluation. If a class is missed, you are responsible for obtaining all class notes and assignments, and you are expected to be fully prepared for the next class session. Except in cases of emergency, there will be no rescheduling of conferences. I understand that situations do occur which are beyone anyone's control; if you must miss class because of such a situation, please notify me by the end of the previous day.

Textbook

This course will be taught using the open source textbook *Calculus, Volume 3*, from the Openstax Project, which is available free online at https://openstax.org/details/calculus-volume-2. This textbook is as uninspiring as the typical calculus textbook (sorry!), but it has the advantage of being free. You can read the textbook online or download a PDF.

Reading

Specific reading assignments will given on the course page on MySLC. The evening after each class, I will post the reading assignment for the next class. Most readings will come from the online textbook. The readings will cover the material we will discuss in the next class. I may also provide links to videos to go along with reading. These are intended to cover the same material as the reading, for those who prefer learning that way. Since I do not plan to lecture, I expect you to have done the reading and/or watched the relevant videos that are assigned for each class. That way we can spend our time together doing problems, clarifying concepts that may have been confusing, etc.

From time to time, I will ask you either watch a YouTube math video or read a short article that I post on MySLC. Since these will be used as the basis for class discussion, **I expect all of you do read these assignments or watch these videos**. This is supposed to be fun! The readings will represent the absolute best of expository mathematics writing, and the videos will be from the best channels I know.

Course Outline

We will cover (roughly) the following topics in this course, in (roughly) the following order.

- Unit 0: Brief review of Calculus 1
  - Limits, derivatives and integrals
  - Exponentials and logs (1.6)
- Unit 1: Sequences and series
  - Sequences and series (5.1, 5.2)
  - Divergence and integral tests (5.3)
  - Improper integrals (part of 3.7)
  - Comparison tests (5.4)
  - Alternating series (5.5)
  - Ratio and root tests (5.6)
- Unit 2: Power series and Taylor series
  - Power series (6.1, 6.2)
  - Taylor series (6.3, 6.3)
- Unit 3: Techniques and applications of integration
  - Integration by Substitution (1.5)
  - Integration by parts (3.1)
  - Partial fractions (3.4)
- Unit 4: Applications to geometric problems
  - Volumes of revolution (2.3)
  - Arc length and surface area (2.4)
  - Parametric curves (7.1, 7.2)
  - Polar coordinates (7.3, 7.4)

Additional topics, such as modeling with elementary differential equations, may be covered if time allows. These may be chosen based on your interest.

Assignments

I will assign a few written problems after each class. These will be posted in the evening after each class, and will be due the next class. Please do these problems on actual paper (not electronically!) and bring them to class on the due date. We will discuss some of these problems during class. I will collect them at the end of each class. I will return them (and we will discuss them) during conference. Late assignments will (in general) not be accepted, since we will have already gone over the problems in class.

These problems are not intended to be busywork. Learning mathematics, like getting to Carnegie Hall, is all about practice, practice, practice. For all assigned problems, I want you to **show all of your work**. The process is as important as, if not more important than, the correct answer. Journey before destination. **You are encouraged to discuss assignments with other students, but you must write up your own solutions independently. When you have collaborated with other students, please acknowledge this by adding a note such as "I discussed question X with person A and person B."** If you use any electronic resources, please indicate that as well. Identical or clearly copied assignments will be treated as violations of academic integrity. Do not postpone the problems until the last minute, as some answers may not come to you immediately, but then become clear a day later. If you are stuck on a problem, feel free to discuss the problem with other students or email me. If you are not able to complete a question, write a short note to describe what you tried and what you think may be important.

Final project

As part of the course, you will explore in depth a topic of your own choosing in conference. The only requirements are that is must be related to calculus (i.e. it must be mathematically based and capture some "sense of the infinite"), must have sufficient depth to warrant a semester's worth of exploration, and must be something you are genuinely interested in. A writeup of your project is due at the end of the course. In addition, you will give a brief presentation (no more than 10 minutes) of your project to the class, which will take place during the final two class periods. I will help you prepare your presentation. In addition, I will make sure that everyone's project is unique, i.e. no two students can work on the exact same thing. I will help you come up with a topic for exploration during your first conference, but please come to that conference having done a bit of brainstorming. Creativity is strongly encouraged. Some ideas you could pursue:

1. Mathematical model of ecological collapse due to insect invasion
2. Mathematical modeling of biochemical reactions
3. Mathematical models of other physical or social phenomena
4. Fractals
5. Harmonics, overtones, and musical timbres
6. Approximating integrals numerically (for ones you can't compute exactly)

The only limit is your imagination... and one semester's time.

Skills assessments

There will be skills assessments after Units 1, 2 and 4, which will be conducted in class. To make sure you have plenty of time to study, these will occur **one week after we finish the corresponding unit**. These will look like exams, but with an important difference. Rather than lose

points for mistakes, you will earn points by demonstrating your proficiency in the skill being tested. The exact skills being tested will be given ahead of time, so you will know what to expect. Each skill will be assessed using the following scale:

4 Excellent
3 Good
2 Basic
1 Improving

While you are not permitted to use your class notes, textbook, or any other references for the skills assessments, you are encouraged to use a **study sheet** for each assessment. You must make the study sheet yourself. You write whatever you want on two sides of a standard piece of paper, and bring it with you to use as a reference during the assessment. After you take the skills assessment, you will have an opportunity to **retest** skills on which you did not perform as well as you may have liked.

Evaluations

The primary form of evaluation at SLC is a written report detailing the progress each student has made over the course of the semester. Your narrative evaluation will each the following:

- Assessments for the specific skills you have acquired in the course, as evaluated in the skills assessments.
- Your final project (written and in-class presentation)
- An overall assessment of your performance in the course

In addition, I am required to give you a letter grade for the course, which will reflect your understanding of the course material, your final project (both written report and in-class presentation), and the effort you have put into the course. We will discuss your progress in the course during conference several times throughout the semester, together with what the appropriate letter grade would be for your current progress.

Self evaluation

You will be required to write a comprehensive self-evaluation at the end the course. This is a both an honest introspection of your performance in this course, as well as an opportinuty to reflect on what you have learned over the semester.

Laptop policy

Please bring your laptop or tablet to every class. Mathematics is highly visual in nature, and we will be using online tools such as Desmos and Wolfram Alpha on a regular basis in class.

That being said, I request that you keep your laptop or tablet closed when you are not using it to do math (unless you are using it to take notes electronically, that's fine!) I request that during class you refrain from using portable electronic devices for any non course-related purposes. I know that math cannot compete with TikTok and Snapchat for your attention, but I will do the best I can to make the time we have together as useful as possible.

Other resources:

For additional help, please take full advantage of the Mathematics Resource Center (MRC). In addition, you can check out the following text and video resources. Topics may differ, and may be covered in different order.

- Paul's online math notes.
- Videos from Trevor Bazett.
- Math TV with Professor V.
- Videos from Krista King.
- Videos for calculus with Dr. Marchese.

Communication

Email is the best way to reach me. During the week, I will try to respond within 24 hours. Email responses may be slower on the weekends, but I will try to reply by Sunday evening. For complex questions, I may ask you to talk with me during conference or after class.

Diversity Statement

While mathematics, in its idealized form, is objective, the practice and teaching of mathematics is not immune from social issues of race, gender, disability, nationality, and socioeconomic status. Pop culture and the media have not made this any better, from Teen Talk Barbie telling us that "math class is tough" to mathematicians and scientists being portrayed in movies and television as "nerdy white males" such as Tony Stark. I will use the following four axioms by Federico Ardila-Mantilla (San Francisco State University) as a foundation for our time together.

- Axiom 1. Mathematical talent is distributed equally among different groups, irrespective of geographic, demographic, and economic boundaries.
- Axiom 2. Everyone can have joyful, meaningful, and empowering mathematical experiences.
- Axiom 3. Mathematics is a powerful, malleable tool that can be shaped and used differently by various communities to serve their needs.
- Axiom 4. Every student deserves to be treated with dignity and respect.

It is my intent that students from all diverse backgrounds and perspectives be well-served by this course. I am committed to a climate of mutual respect both inside and outside of the classroom. To that end,

- I want you to feel comfortable to ask any question you want, to ask me for clarification, or to ask me to slow down, both in class and in office hours. There are no bad questions!
- I want you to feel comfortable "making mistakes" when participating in class. There will be no judgment on my part. I make mistakes all the time! It is a natural part of the learning process.

- If you feel like your performance in the class is being impacted by your experiences outside of class, please do not hesitate to come and talk with me.
- If something was said in class (by me or anyone else) that made you feel uncomfortable, please talk to me about it.
- If you have a name and/or set of pronouns that differ from those that appear in your official records, please let me know.
- If you have any learning differences that you believe will affect your performance in this class, please come talk to me about how best to accommodate these.
- Anonymous feedback may be provided at any point using an anonymous feedback form. I will place a link to this form on MySLC.

Academic Integrity

Violations of community standards of academic integrity will not be tolerated. Please review the Undergraduate Policy on Academic Integrity in the Sarah Lawrence College Student Handbook. Please also note that plagiarism is not just the intentional, but also the unintentional use of another's words or ideas without proper attribution. If you find yourself at any point confused about proper citation format, please ask!

Reasonable Accommodations

If you have a disability that may interfere with your ability to participate in the activities, coursework, or assessment of the objectives of this course, you may be entitled to accommodations. Under the Americans with Disabilities Act and Section 504 of the Vocational Rehabilitation Act of 1973, all students, with or without disabilities, are entitled to equal access to the programs and activities of Sarah Lawrence College. For any questions, please contact the Director of Access and Disability Services, Dan Chan `dchan@sarahlawrence.edu`, 914-395-2235, located in Westlands, first floor.